CABRI AS A *SHARED WORKSPACE* WITHIN THE PROVING PROCESS

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This paper will discuss some findings from a study investigating the development of the proving process in a dynamic geometry environment. Through a detailed analysis of students' processes when working with open geometry problems involving conjecturing and proving in Cabri, an analytical and explanatory framework has been developed. This paper examines in particular the interactions between the students in the proving process. The analysis shows that Cabri works as a shared workspace, i.e. as a space which supports the interaction between students and the construction of shared knowledge in the proving process.

INTRODUCTION AND THEORETICAL FRAMEWORK

The study described in the paper sits within the current mathematics education research strand dealing with the teaching and learning of proof in the context of dynamic geometry environments (e.g. Hoyles & Healy, 1999; Jones, 2000; Laborde, 2000; Mariotti, 2000). In particular, the overall aim of the project (Olivero, 2002; Olivero, Paola, & Robutti, 2001) was to investigate the processes involved in constructing conjectures and proofs in geometry (i.e. in the proving process), when interacting with Cabri, with a particular focus on two things: the interplay between the spatio-graphical field (including Cabri objects, paper drawings, etc) and the theoretical field (including geometrical properties, theorems and definitions) (Laborde, 1998); the interactions taking place in the proving process, both between students and between the students and the tools used (mainly Cabri). This paper will focus on the second issue.

A number of studies (e.g. Crook, 1994; Kieran & Dreyfus, 1998) deals with issues relating to the interactions between subjects working together both with and without computers. When two students work together on the same problem, it must not be taken for granted that they can automatically communicate and really 'work together'. "The process of collaborative learning is not homogeneous or predictable, and does not necessarily occur simply by putting two students together" (Teasley & Roschelle, 1993, p.253). Individuals must make a continuous effort to coordinate their language and activity with respect to shared knowledge and to construct a Joint Problem Space (Teasley & Roschelle, 1993). Boero et al. (1995) introduced the construct of "field of experience", defined as "the system of three evolutive components (external context, student internal context, teacher internal context) referred to a sector of human culture which the teacher and students can recognise and consider as unitary and homogeneous". The notion of internal and external contexts relates to the construction of a shared workspace. The internal context[1] is what is and happens in the mind of the students, while

[1] The notion of internal context is based on the definition of context given by Edwards & Mercer (1987): "We shall use the term *context* to refer to everything that the participants in a conversation know and

the external context is what is produced and visible (as for example Cabri figures, statements, etc)[2]. When working together students need to make their internal contexts explicit in order to be able to communicate.

This paper will discuss the interactions taking place between the students when working at the computer within the proving process. First, the methodology of the research will be sketched; second, typologies of students' interactions will be defined and examples from students' work explored; finally, a preliminary model interpreting these interactions will be presented.

METHODOLOGY

The research consisted of classroom interventions which took place in a number of secondary schools (15-17 years old pupils) in England and Italy. Students were asked to solve open problems in geometry, working in pairs and using Cabri. Within the classroom interventions, observations of case studies of pairs of students were carried out. The methods used were video-recording and collection of material. The data available for the analysis were transcripts from the video-tapes, the Cabri files and the students' worksheets.

The two extracts discussed in the following sections are taken from the work of Bartolomeo and Tiziana, solving the problem Perpendicular bisectors of a quadrilateral[3]. The students[4] are 15 years old and belong to a second year classroom of a Liceo Scientifico in Turin (Italy).

STUDENTS INTERACTING *THROUGH* CABRI: A RESEARCH PROBLEM

When there are two (or more) students solving the problem *at* the computer, everyone has his/her own internal context. How can students communicate and share their understanding? The construction of a Joint Problem Space (Teasley & Roschelle, 1993) was considered a relevant category of analysis in the proving process (Olivero, 2002) as it may either support or get in the way of the evolution of the proving process itself.

When two students are asked to solve a problem together *at* the computer, each student tells his/her own story, but at the same time the two stories need to intersect, given that only one computer is available to them. What can be noticed is that there are moments in the process in which the students think and do different things and moments in which a good communication takes place and the students really work on the 'same' problem. An interesting thing is to observe how students get to communicate and 'merge' their stories towards a common goal, which is the production of conjectures and proofs. This will be the focus of this paper.

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understand, over and above that which is explicit in what they say, that contributes to how they make sense of what is said" (p.63).

[2] This research did not take into account the teacher internal context.

[3] You are given a quadrilateral ABCD. Construct the perpendicular bisectors of its sides: a of AB, b of BC, c of CD, d of DA. H is the intersection point of a and b, K of b and c, L of c and d, M of a and d. Investigate how HKLM changes in relation to ABCD. Prove your conjectures.

[4] They are medium achievers with respect to mathematics. They had already used Cabri some times before this session, working on construction and exploration problems.

Bartolomeo and Tiziana have shown moments in which communication was taking place between them and moments in which they seemed to be focused on different things and not really communicating between each other. Two types of interaction were identified and the process of occurrence of each one and of transition from one to the other is revealed to be interesting for the development of the proving process. The different occurrences of the ‘interaction’ category (Olivero, 2002) are presented in the following table.

Synchronous interaction	The two students 'see' the same thing on the same figure. The discourse is spoken by the two together, helping each other, interrupting each other.
Asynchronous interaction	The two students look at the same figure but focus on different aspects, say different things, follow different solution strategies. One student has got the mouse and the other one wants it. Indicators are for example ‘what are you doing!’, ‘why did you do that?’, ‘wait!’.

In the following, two episodes showing different types of interaction between the students are presented.

Asynchronous interaction: two ways of 'seeing' the same dragging episode

In this extract, the same episode of dragging is 'seen' in two different ways. This is seen as an indicator of asynchronous interaction.

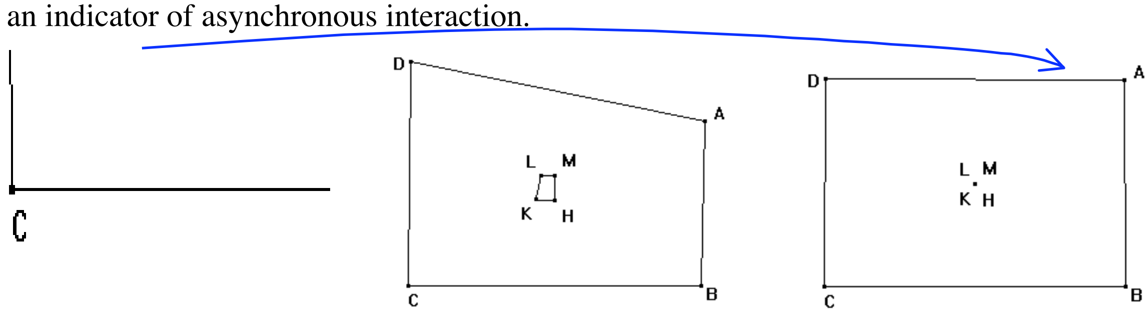

Figure 2 Figure 3 Figure 4

69 Meanwhile Tiziana drags ABCD into a rectangle (Figure 2)
70 **Bartolomeo:** what have you done, **a rectangle?**
71 Tiziana: yes, well…
72 **Bartolomeo:** **so… it is a point… try to make it bigger…**
75 **Tiziana drags D up and stops** to observe and think (Figure 3)
76 **Tiziana:** **excuse me! This (she points at LM) follows what this (AB) does, this (LK) follows this (AD) …** (she laughs)
77 Bartolomeo: let’s examine some more cases
78 Tiziana drags A up and gets Figure 4
79 **Bartolomeo:** **ah, when it’s a rectangle it’s always a point…** (he writes down the second conjecture) … if… shall I write “disappears” or “is a point”? It's a point…
80 **Tiziana:** **No, because now it's a point too.**

Tiziana drags ABCD to a rectangle (Figure 2), without saying anything. Bartolomeo does not follow Tiziana and does not understand why she made a rectangle (70), but then he

tries to work on what he is presented with, observing that "it is a point" (72). Tiziana does not communicate her ideas to Bartolomeo. Given that she has the mouse, she can do what she wants. Having the mouse allows her to control the situation autonomously. Bartolomeo follows her (70-80) until he needs to have the mouse himself (immediately after) in order to get control again over the situation. In fact his strategy is that of checking all the particular cases, while Tiziana is more open to what Cabri shows her. However, even if Bartolomeo and Tiziana are working on the same figure and the same case (ABCD rectangle), what they 'see' is different. They are seeing the figure on the screen and relating it to their two different internal contexts. And this is shown by the use of dragging in particular. Bartolomeo sees than when Tiziana gets a rectangle then HKLM is a point. He wants to do a sort of dragging test (72) that seems to be a kind of test at a perceptual level ("make it bigger"-72): the aim is to check if it is still a point in another rectangle case. However Tiziana, who has the mouse, does what she wants, moving from Figure 2 to Figure 4 via Figure 3. She stops in 75 and reads a relationship between elements of the figure (she sees a relationship between the side of ABCD and of HKLM, which will be transformed into a conjecture later on in the process). She expresses her reasoning, but Bartolomeo does not follow her. He is thinking about his conjecture. He pays attention only to the initial and final figure (Figure 2 and Figure 4), as two snapshots, as his aim was clear: checking if HKLM is always a point when ABCD is a rectangle. As soon as Tiziana stops in Figure 4, Bartolomeo formulates the conjecture for the rectangle (79). After this, Tiziana seems to abandon her line of thoughts and follows Bartolomeo, and in 80 she shows that she is thinking about the rectangle case, even if in a different way (“no, because now it’s a point too”).

Towards a synchronous interaction: the 'space' of the parallelogram.

The following extract starts with a situation in which the two students are thinking about and doing different things. Bartolomeo, who does not have the mouse and therefore cannot do what he wants, suggests an idea, but Tiziana, who has the mouse, chooses to do something else. However, the whole episode converges to a communication within the shared Cabri space.

49	**Bartolomeo:**	Now go to pointer and **let’s try to move…** […]
52	Tiziana drags D randomly rightwards and then leftwards (Figure 5)	
53	Bartolomeo:	so, let’s do this…
54	**Tiziana:**	eh, excuse me, **isn’t that a…** […]
57	**Bartolomeo:**	**ok, try to make it a trapezium…**
58	Tiziana drags D	
59	Tiziana:	is it a trapezium?
60	**Bartolomeo:**	**let’s see what happens in every case**, shall we?
61	**Tiziana:**	**wait, eh… let’s do this…**
62	Tiziana drags C	
63	**Bartolomeo:**	**… a parallelogram?**
64	Tiziana stops moving when she gets a parallelogram (Figure 6)	
65	**Bartolomeo:**	**ok, so… if ABCD is a parallelogram, then…**
66	**Tiziana:**	**this is a parallelogram too**

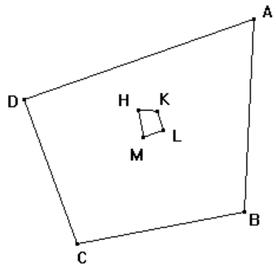

Figure 5

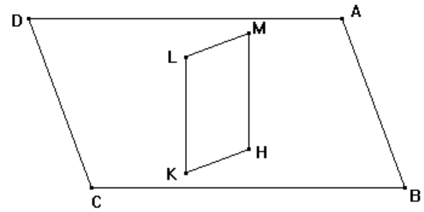

Figure 6

Bartolomeo has a strategy in mind and wants to move ABCD (49). Tiziana starts moving in 52, accomplishing Bartolomeo's wish. Bartolomeo continues to think about his own strategy (53), which he has not made explicit yet. Tiziana is thinking about something on her own. She is looking at the Cabri figure and 'reading' it (54). Bartolomeo makes explicit what he wants to do (57). Tiziana follows what Bartolomeo says but without seeming to understand where he wants to go (58). Bartolomeo makes explicit the general strategy he has in mind, that is an ordered exploration of cases (60): what happens to HKLM when ABCD is a…? This time Tiziana does not pay attention to what Bartolomeo says and pursues her own idea ("wait"-61), without talking Bartolomeo through it. They are going along two different paths. They both have ideas so it is the person who has the mouse that leads the situation, i.e. Tiziana; she does what she wants (62), forcing Bartolomeo to follow her in her thoughts. Bartolomeo is surprised to see a parallelogram (63) on the screen because he does not know what Tiziana wants to achieve. It is only when dragging is stopped (64) that the two students produce a conjecture about the same thing. At this point (65-66) the students seem to be sharing the 'same' story, after a whole episode in which they were not communicating (49-64). The communication seems to take place around Figure 5 and it seems to be more a fact of 'tuning' each other's thoughts than only communicating with each other. It is a visual element, not a spoken one that provides mutual sharing and understanding. There is no need of speaking at this point, everything happens around a Cabri figure which now has a shared meaning: synchronous interaction is taking place and a conjecture can be produced by both students at the same time (65-66).

BUILDING A SHARED WORKSPACE AND SYNCHRONISING INTERNAL CONTEXTS

This section elaborates on the previous extracts, providing a preliminary model interpreting students' interactions in the proving process.

When two students work together at the computer solving an open problem, the internal context of each student is projected into the problem situation. Using a metaphor, this projection forms two different 'shadows' in the external context. As Figure 7 shows, at the beginning of the process it is likely that the two 'shadows' do not intersect, as the students' internal contexts may be different. The starting point may be a state of asynchrony. During the process there is a continuous feedback from the external context to students' actions, therefore the internal context is constantly modified, and a process of

"internalisation[5]" (Vygotsky, 1978, p.56) takes place, so that the external tools (Cabri tools) may be internalised as psychological tools which direct students' behaviour in solving the problem. The students see things in Cabri and they relate what they see to their internal context, therefore they may see different things on the same figure (see the first extract). They work on what they see and they transform it. They produce statements. But at the same time through the 'shadows' the students start to interact with each other's internal contexts and intentions. A point may be reached in which the projections of the two internal contexts intersect (Figure 8). When this happens it means that the students are communicating and working together on the same issues. Synchronisation may then take place. The internal context is then continuously modified by both the feedback from the Cabri environment and the interaction with the other student. The projection of the internal contexts in the external context should be imagined as dynamic, so that at times the intersection exists and at times it does not. The moments in which the intersection exists are the moments in which there is the construction of joint understanding and knowledge, which may support the production of conjectures and proofs. The moments of synchronisation do not necessarily coincide with the development of well-formed logical statements. In fact it seems that if there is a synchrony between the students they understand each other perfectly through the external space (mainly Cabri) without finding the need of developing a well-formed logical language. Things can be seen and understood in Cabri, without any need for explicit logic and Cabri becomes part of students' interactions. As Teasley & Roschelle (1993) state,

> students are not wholly dependent on language to maintain shared understanding. In fact, one major role of the computer in supporting collaborative learning is providing a context for the production of action and gestures. (p.238)

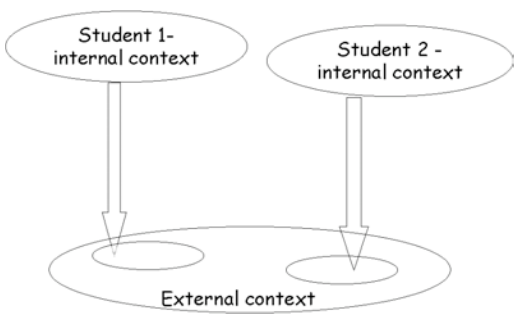

Figure 7. Two internal contexts are projected onto the external context

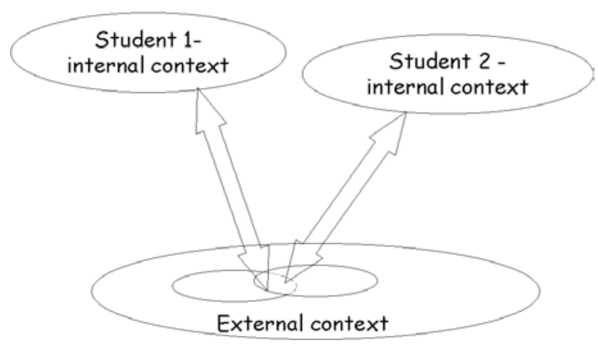

Figure 8. The internal contexts communicate through the external context.

Summarising, the analysis suggests that the Cabri environment is revealed to be a *shared workspace* for students, that is a space in which students communicate and converge

[5] "The process of internalization consists of a series of transformations: (a) An operation that initially represent an external activity is reconstructed and begins to occur internally. […]. (b) An interpersonal process is transformed into an intrapersonal one. […]. (c) The transformation of an interpersonal process into an intrapersonal one is the result of a long series of developments" (Vygotsky, 1978, p.56-57).

towards a shared understanding. In the second extract presented above the shared space was the 'space' of the parallelogram.

> Already A has some insight into the state of B's understandings, the meanings which are evoked for B by the problem or the situation. […] The computer has brought an arena in which A and B's understandings can be externalised. (Noss & Hoyles, 1996, p.5)

Entering another's *universe of thought* (Trognon, 1993) in Cabri may be easier than in paper and pencil, because of the possibility of moving figures on the screen which contrasts with the fact that figures on paper are static. Both the background knowledge and the knowledge being constructed over the solution process can be expressed, changed and explored via dragging in Cabri.

> A has now a language with which to interact with B, the language of action in which ideas on the computer are expressed. […] A and B both have a two-way channel of communication with the computer, and in establishing these channels, it (actually the setting) has opened a channel from B to A where previously the direction of communication was essentially one-way. (Noss & Hoyles, 1996, p.5)

Seeing the dynamic variation of figures on the screen allows interactive participation of both students to the same experience. At the beginning it seems that the one who has got the mouse leads the solution process, however it is observed that once figures start moving on the screen also the other person is allowed to enter the experience and process of discovery, which seems to be a pre-requisite for the construction of shared knowledge or understanding. The possibility of direct manipulation of Cabri objects through the mouse makes Cabri an external space in which the two subjects can interact and communicate, trying to *synchronise their internal contexts*.

CONCLUDING REMARKS

The modalities of students' interactions are defined by what students say and do. The extracts discussed in this paper analysed in particular the different interpretation of dragging and the ownership of the mouse. The students communicate through dragging while interacting in front of the computer and dragging is one of the possibilities students have to make explicit their internal contexts. So their interpretation of dragging[6] affects how the communication is carried out and contributes to the solution of the problem. Other directly 'observable' variables which provide information about students' interaction, and which have been discussed elsewhere, are:

- The different types of interaction of the students with the software and the ways in which the students incorporate (or not) the software in their thinking over the proving process (Olivero, 2002).
- The ways in which they use paper and pencil sketches (when they do) in order to break the interaction and think on their own or to communicate something that cannot be done through Cabri (Olivero, 2002).
- The language they use to communicate *with* and *at* the computer (Arzarello, 2000).

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[6] See Arzarello et al. (1998; 2002) and Olivero (2002) for a detailed classification of students' use of different dragging modalities.

References

Arzarello, F. (2000). Inside and Outside: Spaces, Times and Language in Proof Production. In T. Nakahara & M. Koyama (Eds.), *Proceedings of PME24* (Vol. 1, pp. 23-38). Hiroshima.

Arzarello, F., Gallino, G., Micheletti, C., Olivero, F., Paola, D., & Robutti, O. (1998). Dragging in Cabri and modalities of transition from conjectures to proofs in geometry. In A. Olivier & K. Newstead (Eds.), *Proceedings of PME22* (Vol. 2, pp. 32-39). Stellenbosh.

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in Cabri environments. *Zentralblatt fur Didaktik der Mathematik*.

Boero, P., Dapueto, C., Ferrari, P., Ferrero, E., Garuti, R., Lemut, E., Parenti, L., & Scali, E. (1995). Aspects of the Mathematics-Culture Relationship in Mathematics Teaching-Learning in Compulsory School, *Proceedings of PME19 (*Vol. 1, pp. 151-166). Recife.

Crook, C. (1994). *Computers and the collaborative experience of learning*. London: Routledge.

Edwards, D., & Mercer, N. (1987). *Common Knowledge*. London and New York: Routledge.

Hoyles, C., & Healy, L. (1999). Linking informal argumentation with formal proof through computer-integrated teaching experiments. In O. Zaslavsky (Ed.), *Proceedings of PME23* (Vol. 3, pp. 105-112). Haifa.

Jones, K. (2000). Providing a foundation for deductive reasoning: students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics, 44*, 55-85.

Kieran, C., & Dreyfus, Y. (1998). Collaborative versus individual problem solving: entering another's universe of thought. In A. Olivier & K. Newstead (Eds.), *Proceedings of PME22* (Vol. 3, pp. 112-119). Stellenbosh.

Laborde, C. (1998). Relationship between the spatial and theoretical in geometry: the role of computer dynamic representations in problem solving. In J. D. Tinsley & D. C. Johnson (Eds.), *Information and Communications Technologies in School Mathematics* (pp. 183-195). London: Chapman & Hall.

Laborde, C. (2000). Dynamic Geometry Environments as a Source of Rich Learning Contexts for the Complex Activity of Proving. *Educational Studies in Mathematics, 44*(1/2), 151-161.

Mariotti, M. A. (2000). Introduction to Proof: The Mediation of a Dynamic Software Environment. *Educational Studies in Mathematics, 44*(1/2), 25-54.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings : learning cultures and computers*. Dordrecht ; Boston: Kluwer Academic Publishers.

Olivero, F. (2002). *The proving process within a dynamic geometry environment*. Unpublished thesis, University of Bristol, Bristol.

Olivero, F., Paola, D., & Robutti, O. (2001). Approaching theoretical thinking within a dynamic geometry environment. *L'educazione matematica, 3*(3), 127-148.

Teasley, S. D., & Roschelle, J. (1993). Constructing a Joint Problem Space: the Computer as a Tool for Sharing Knowledge. In S. P. Lajoie & S. J. Derry (Eds.), *Computers as Cognitive Tools* (pp. 229-258). London: Chapman & Hall.

Trognon, A. (1993). How does the process of interaction work when two interlocutors try to resolve a logical problem? *Cognition and Instruction, 11*(3&4), 325-345.

Vygotsky, L. S. (1978). *Mind in society. The development of higher psychological processes.* Cambridge, MA: Harvard University Press.